

**EXPANSION FORCE AS A BOHMIAN QUANTUM EFFECT  
ON ELEMENTARY PARTICLES**

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**Abstract**

The new expansion force, which has been proposed to explain flat rotation curves without Dark Matter, is shown to be a consequence of Bohmian Quantum Mechanics applied to elementary particles in an inverse-square interaction.

This gives a new fundamental validation of this expansion force and opens the way to a new field of experimental verifications.

**KEYWORDS**

Expansion force, gravity, Bohmian Quantum Mechanics, inverse-square interaction

## 1. Introduction

Ten years ago, I proposed a new expansion dynamics paradigm [1], introducing a “cosmic expansion acceleration” to explain the galactic flat rotations curves [2] with unchanged Newton’s gravity and without Dark Matter. This acceleration has been proposed to be proportional to the velocity vector  $\vec{v}$  according to :

$$\vec{\gamma} = \frac{\dot{r}}{r} \vec{v} \quad (1)$$

Where the coefficient  $\frac{\dot{r}}{r}$  is the local expansion rate.

This paradigm has been applied to different phenomena, such as the question of planar structures of satellite galaxies [3] and to large scale cosmic evolutions [4, 5].

Considering a nonhomogeneous radially symmetrical universe, I have shown that this new acceleration can be deduced from the solution of Einstein’s equation, thus being validated in General Relativity [4]. According to this point of view, gravity appears to be a dual process, including the unchanged (inward) Newton’s acceleration plus the outward expansion acceleration (which is very small in common experience). Negative masses are needed for this cosmologic model and the positive and negative mass repartitions have been computed [5].

Meanwhile, the same idea has been reintroduced by a Chinese team [6] with a lagrangian approach (applied to the radial acceleration only, and with a different coefficient in eq. (1)).

Incidentally, when this paradigm is restricted to the radial part only, it can be compared to the MOND approach [7], where the radial acceleration of a circular trajectory is supposed to be modified, through a threshold procedure. The new expansion acceleration has often been wrongly assimilated to MOND, in spite of the two important differences:

- The proposed expansion acceleration (1) is simply added to Newtons’s one, which is not modified.
- It has also a lateral part:  $\gamma_l = \frac{\dot{r}}{r} r \dot{\theta} = \dot{r} \dot{\theta}$  (2)

In fact, this lateral part is absolutely needed in our paradigm, so as to modify the *lateral* velocity of the flat rotation curves [1]. But (unlike MOND), it was not absolutely necessary to add a radial acceleration. It was assumed by induction, to have the same form as the lateral one:

$$\gamma_r = \frac{\dot{r}}{r} \dot{r} = \frac{\dot{r}^2}{r} \quad (3)$$

leading to the vectorial formulation (1).

Up to this point, this new paradigm has not found deep justifications.

In 2015, I proposed that it could be generated by a mass erosion process [8].

More recently, such a vectorial additional acceleration has also been proposed by Maeder [9], based on quite different arguments (scale invariance of the macroscopic empty space).

Meanwhile, I searched for theoretical justifications in Bohmian Quantum Mechanics [10, 11], studying a far-away galaxy as a "particle" submitted to the pilot wave of the total mass of the universe [12]. This approach results in an infinitely expanding and contracting universe (no Big Bang) and allows to deduce the observed variations of the Hubble Constant and acceleration coefficient.

The present contribution is focused on the same Bohmian Quantum Mechanics approach, but at the probably more fundamental level of elementary particle interactions.

## 2. Bohmian Quantum interaction of two elementary particles

Let-us consider a couple of two identical microscopic particles submitted to a central inverse-square interaction (which could be gravitational or electromagnetic). In a first step, we restrict their movement to a 1D. non-rotating straight line space. Each particle (supposed to have the same mass  $m$ ) can be considered, in the mass center system, to be submitted to a potential energy, noted as:

$$V = -m \frac{A}{r} \quad (4)$$

where  $r$  represents the distance between the two particles and  $A$  is a constant (positive for gravity, but a-priori, it could also be negative).

Each particle has a mass  $m$  and, due to the reduced mass  $\frac{m}{2}$ , their acceleration in the central mass system is equal to  $-\frac{2A}{r^2}$ .

The pilot wave associated to each particle is supposed to be Gaussian. In order to satisfy Schrödinger's equation and normalization, it must be written, in amplitude  $R$  and phase  $\frac{S}{\hbar}$  as [13]:

$$\psi(r, t) = R(t) e^{i \frac{S(r, t)}{\hbar}} \quad \text{with} \quad \frac{S}{\hbar} = \frac{r^2}{4\Delta^2 \left(1 + \frac{t^2}{\tau^2}\right) \tau} - \frac{vt}{\hbar} \quad (5)$$

where  $\Delta$  is the width of the initial wave packet and  $\tau$  represents the time scale parameter:

$$\tau = \frac{2m\Delta^2}{\hbar} \quad (6)$$

According to the pilot wave theory [10, 11], the particle velocity is obtained by:



$$m \frac{dr}{dt} = \frac{\partial S}{\partial r} = \left( \frac{\hbar r}{2\Delta^2 \left(1 + \frac{t^2}{\tau^2}\right) \tau} - \frac{\partial V}{\partial r} \right) t \quad (7)$$

For an elementary particle, such as an electron, the time-scale  $\tau$  is extremely small (equal to  $10^{-26}$ s. for  $\Delta \sim 10^{-15}$ m.). At this point, our problem differs from the cosmic evolution of a galaxy, where  $\tau$  was extremely high [12].

Now, from (6) and  $t \gg \tau$ , eq. (7) can be approximated by:

$$m \frac{dr}{dt} = m \frac{r}{t} - m \frac{At}{r^2} \quad (8)$$

This equation can be solved (thru the variable change  $u = \frac{r}{t}$ ), resulting in:

$$r^3 = 3At^2 + Ct^3 \quad (9)$$

where C is a constant of integration.

For  $A > 0$  (attractive potential, gravity) a positive value of C is needed for large time. The relative weight of the two right-side terms in eq. (9) depend on the value of:

$$t_0 = \frac{3A}{C} \quad (10)$$

which is very small for common velocities ( $A = mG \sim 10^{-40}$  for an electron). In this case, a quasi-linear movement is rapidly obtained.

For zero energy particles, the velocity at infinity tends to zero. This implies  $C = 0$ , and eq. (9) becomes:

$$r^3 = 3At^2 \quad (11)$$

[For this equation, free fall is represented by  $t$  going from  $-\infty$  to 0.]

Successive derivation allows to compute  $\dot{r}$  and  $\ddot{r}$ :

$$r^2 \dot{r} = 2At \quad (12)$$

Noting that, from (11) and (12):

$$\frac{\dot{r}^2}{r} = \frac{4}{3} \frac{A}{r^2} \quad (13)$$

the acceleration can be finally written as:

$$\ddot{r} = -\frac{2A}{r^2} + \frac{\dot{r}^2}{r} \quad (14)$$

The first term on the right is the classical inverse-square acceleration. The second term represents the additional expansion acceleration:



$$\gamma_r = \frac{\dot{r}^2}{r} = \frac{\dot{r}}{r} \dot{r} \quad (15)$$

This radial acceleration is clearly a consequence of Bohmian Quantum dynamics applied to the particles and their pilot waves.

### 3. Where the lateral part comes from

Let-us now suppose that the preceding straight line is turning around a perpendicular central axis, generating a planar trajectory. We choose the particular angular velocity:

$$\Omega = \frac{v_0}{r} \quad (16)$$

Where  $v_0 = r\dot{\theta}$  has been chosen to be a constant lateral velocity in order to represent the flat rotation curve observations.

Due to the acceleration transformation law between the initial line space and the rotating line, the two following lateral terms (usually called tangential and Coriolis terms) must be added to the acceleration:

$$\vec{\gamma}_l = \frac{\partial \vec{\Omega}}{\partial t} \wedge \vec{r} + 2\vec{\Omega} \wedge \frac{\partial \vec{r}}{\partial t} \quad (17)$$

In the case of the rotation velocity (16), it is easy to show that:

$$\gamma_{l=} \frac{\dot{r}}{r} v_0 \quad (18)$$

(the - non mentioned here - centrifugal term  $\frac{v_0^2}{r}$  does also appear as usual).

Eq. (18) is nothing else than the lateral additional expansion acceleration to be added in order to guarantee the flat rotation curve.

This result confirms our initial choice of an expansion acceleration proportional to the vectorial velocity and local expansion rate, as it is clear for the lateral part (eq. (18)) and for the additional radial part (eq.15).

Finally, the lateral part (which explains flat rotation curves) simply results from the rotation dynamics, whenever the radial part is a consequence of Bohmian Quantum Mechanics applied to the considered particles.

### 4. Comments on Universe expansion and possible experimental verifications

Of course, this very simple 2 particle interaction model cannot predict by itself what happens for the expansion of the Universe. However, for instance, it can be envisioned that – in the case of a homogeneous infinite Universe – the Newton's contribution tends to be strongly reduced, due to symmetric cancellations. In this case, eq. (14) should be reduced to its expansion term, leading to a quasi-exponential expansion.

More complex configurations would produce various expansion dynamics, depending on the mass repartition in the Universe.

Going back to our simple 2 particle model with a far-away zero velocity, the expansion rate can be easily computed from (12):

$$\frac{\dot{r}}{r} = \frac{2}{3t} \quad (19)$$

It is notable that this does not depend on  $A$ : the expansion force produces a natural repulsive tendency for particles, even when they are attractive. And, according to eq. (13), the closer are the two particles, the more important is the repulsive expansion force.

Another interesting result is that, from (11), the second derivative can also be written as:

$$\ddot{r} = -\frac{2A}{3r^2} \quad (20)$$

So, when compared to pure Newton's gravity, the "Bohmian free-fall" of a zero energy particle just happens as if  $A$  was divided by 3. Practically, it can be seen from (12) that, at a given place in the fall ( $t$  and  $r$  given), the local velocity should be divided by 3 for the Bohmian free fall, due to the opposite effect of the expansion force. This can be a possible benchmark for future experimental verifications.

More generally, eq. (9) is the one to be verified. The possibility of a non-zero constant velocity  $v_\infty$  at infinite time leads to  $C = v_\infty^3$ . In this case, due to the third degree of the curve (9), a bounce effect should be observed (where  $\dot{r} = 0$ ) at time  $t_B = -\frac{2A}{C}$  and radius:

$$r_B = \sqrt[3]{4} \frac{A}{v_\infty^2} \quad (21)$$

In the far-away zone, simple developments lead to

$$\dot{r} - v_\infty \simeq \frac{A^2}{v_\infty^3 r^2} \quad (22)$$

to be compared with the equivalent development for a purely Newtonian interaction:

$$\dot{r} - v_\infty \simeq \frac{A}{v_\infty r} \quad (23)$$

Due to the  $r^{-2}$  variation in (22), the fall's acceleration is of course reduced in case of a (repulsive) expansion, and this reduction is more important for large velocities. Furthermore, for the second derivative (deduced from (22), a  $r^{-3}$  variation is obtained for large  $r$ :

$$\ddot{r} = -2 \frac{A^2}{v_\infty^2 r^3} \quad (24)$$

Which can be another benchmark for future experimental verifications.



Let-us recall that our paradigm concerns not only gravity but also any inverse-square force, since  $A$  can be negative. Comparable observations can be made in this case (no rebound effect happens in this case, due to the negative value of (21)).

## 5. Conclusion

Our present results confirm the proposal of a “fifth force” as a consequence of Bohmian Quantum Mechanics, since it has been found to be there at the cosmic level of galaxy expansions [12] and here, at the more fundamental level of elementary particles. As it was originally stated [1, 8], it is a general dynamics principle, assumed to be valid not only for the problem of flat rotation curves. Incidentally, it seems to be there for any central inverse-square - attractive or repulsive – force: gravity and electromagnetism.

The MOND radial modification is equivalent to an additional attractive acceleration, which can be thought of as due to an added positive (dark) matter. According to this point of view, dark matter is nothing else than a mathematical equivalence of a ad-hoc radial gravity modification.

In our paradigm, the added radial (expansion) acceleration is equivalent to additional negative matter. Since anti-matter does exist, it seems that it could be more promising to seek negative matter [12, 14] rather than dark matter, even though its repartition in the universe is not well understood. Up to now, the negative mass repartitions corresponding to the expansion force has been computed for the simple case of nonhomogeneous symmetrical universes [5, 12]. These simulations suggest that positive masses do predominate in our surroundings, while there should be an excess of negative masses in far-away regions [12, 14]. This could be in conformity with the present observations of large-scale regions.

Furthermore, this mass repartition can be related to the spatial evolution curve of the Hubble Constant [12], which gives another benchmark for future verifications related to the present data from the Hubble constant measurements.

These preliminary results will have to be confronted to observations and/or simulations in order to be accepted or not as a support for reality. Our present contribution increases the field of experiences and simulations to be done to the domain of elementary particles.



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